



Universidade Federal de São Carlos  
Departamento de Engenharia de Produção



# Otimização Linear Contínua e Discreta

PPGEP, UFSCar - Semestre 01/2025  
Prof. Dr. Pedro Munari (munari@dep.ufscar.br)

Tópico 6.1: Métodos de pontos interiores: ideia e um pouco de história

# Apresentação

## Objetivos desta semana:

- ▶ Conhecer os métodos de pontos interiores, sua história e a variante primal-dual;
- ▶ Entender os conceitos de barreira logarítmica, trajetória central, vizinhança e passo de Newton;
- ▶ Compreender o algoritmo do método primal-dual de pontos interiores e usá-lo na resolução de problemas de programação linear.

# Apresentação - Tópico 6.1

## Objetivos deste tópico:

- ▶ Entender a ideia dos métodos de pontos interiores obtidos a partir do KKT;
- ▶ Conhecer a história dessa classe de métodos.

## Vídeo-aula:

- ▶ <https://www.youtube.com/watch?v=5F0MoJMuVhI>

# Métodos de pontos interiores

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$$\begin{aligned}Ax &= b \\ A^T p + s &= c \\ x_j s_j &= 0, \quad j = 1, \dots, n \\ x, s &\geq 0\end{aligned}$$

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  - **Método primal-dual:** usa (1) e (2) simultaneamente para calcular as direções de busca.

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- ▶ 1979: Khachyan propôs o método Elipsóide;
- ▶ Embora com complexidade polinomial, o método elipsóide é extremamente lento na prática;
- ▶ O método de Karmarkar foi o primeiro algoritmo com complexidade polinomial que funcionou bem na prática.

# O algoritmo de Karmarkar...

[www.nytimes.com/1984/11/19/us/breakthrough-in-problem-solving.html](http://www.nytimes.com/1984/11/19/us/breakthrough-in-problem-solving.html)

## *Breakthrough in Problem Solving*

By JAMES GLEICK

A 28-year-old mathematician at A.T.&T. Bell Laboratories has made a startling theoretical breakthrough in the solving of systems of equations that often grow too vast and complex for the most powerful computers.

The discovery, which is to be formally published next month, is already circulating rapidly through the mathematical world. It has also set off a deluge of inquiries from brokerage houses, oil companies and airlines, industries with millions of dollars at stake in problems known as linear programming.

### **Faster Solutions Seen**

These problems are fiendishly complicated systems, often with thousands of variables. They arise in a variety of commercial and government applications, ranging from allocating time on a communications satellite to routing millions of telephone calls over long distances, or whenever a limited, expensive resource must be spread most efficiently among competing users. And investment companies use them in creating portfolios with the best mix of stocks and bonds.

The Bell Labs mathematician, Dr. Narendra Karmarkar, has devised a radically new procedure that may speed the routine handling of such problems by businesses and Government agencies and also make it possible to tackle problems that are now far out of reach.

"This is a path-breaking result," said Dr. Ronald L. Graham, director of mathematical sciences for Bell Labs in Murray Hill, N.J.

"Science has its moments of great progress, and this may well be one of them."

Because problems in linear programming can have billions or more possible answers, even high-speed computers cannot check every one. So computers must use a special procedure, an algorithm, to examine as few answers as possible before finding the best one — typically the one that minimizes cost or maximizes efficiency.

A procedure devised in 1947, the simplex method, is now used for such problems,

**Continued on Page A19, Column 1**

# O algoritmo de Karmarkar...

<http://content.time.com/time/magazine/article/0,9171,923774,00.html>



Karmarkar at Bell Labs: an equation to find a new way through the maze

## Folding the Perfect Corner

*A young Bell scientist makes a major math breakthrough*

**E**very day 1,200 American Airlines jets crisscross the U.S., Mexico, Canada and the Caribbean, stopping in 110 cities and bearing over 80,000 passengers. More than 4,000 pilots, copilots, flight personnel, maintenance workers and baggage carriers are shuffled among the flights; a total of 3.6 million gal. of high-octane fuel is burned. Nuts, bolts, altimeters, landing gears and the like must be checked at each destination. And while performing these scheduling gymnastics, the company must keep a close eye on costs, projected revenue and profits.

Like American Airlines, thousands of companies must routinely untangle the myriad variables that complicate the efficient distribution of their resources. Solving such monstrous problems requires the use of an abstruse branch of mathematics known as linear programming. It is the kind of math that has frustrated theoreticians for years, and even the fastest and most powerful computers have had great difficulty juggling the bits and pieces of data. Now Narendra Karmarkar, a 28-year-old

Indian-born mathematician at Bell Laboratories in Murray Hill, N.J., after only a years' work has cracked the puzzle of linear programming by devising a new algorithm, a step-by-step mathematical formula. He has translated the procedure into a program that should allow computers to track a greater combination of tasks than ever before and in a fraction of the time.

Unlike most advances in theoretical mathematics, Karmarkar's work will have an immediate and major impact on the real world. "Breakthrough is one of the most abused words in science," says Ronald Graham, director of mathematical sciences at Bell Labs. "But this is one situation where it is truly appropriate."

Before the Karmarkar method, linear equations could be solved only in a cumbersome fashion, ironically known as the simplex method, devised by Mathematician George Dantzig in 1947. Problems are conceived of as giant geodesic domes with thousands of sides. Each corner of a facet on the dome

# O algoritmo de Karmarkar...

## THE STARTLING DISCOVERY BELL LABS KEPT IN THE SHADOWS

Now its breakthrough mathematical formula could save business millions

It happens all too often in science. An obscure researcher announces a stunning breakthrough and achieves instant fame. But when other scientists try to repeat his results, they fail. Fame quickly turns to notoriety, and eventually the episode is all but forgotten.

That seemed to be the case with Narendra K. Karmarkar, a young scientist at AT&T Bell Laboratories. In late 1984 the 28-year-old researcher astounded not only the scientific community but also the business world. He claimed he had cracked one of the thorniest aspects of computer-aided problem-solving. If so, his feat would have meant an instant windfall for many big companies. It could also have pointed to better software for small companies that use computers to help manage their business.

Karmarkar said he had discovered a quick way to solve problems so hideously complicated that they often defy even the most powerful supercomputers. Such problems bedevil a broad range of business activities, from assessing risk factors in stock portfolios to drawing up production schedules in factories. Just about any company that distributes products through more than a handful of warehouses bumps into such problems when calculating the cheapest routes for getting goods to customers. Even when the problems aren't terribly complex, solving them can chew up so much computer time that the answer is useless before it's found.

**HEAD START.** To most mathematicians, Karmarkar's precocious feat was hard to swallow. Because such questions are so common, a special branch of mathematics called

twist. Other scientists weren't able to duplicate Karmarkar's work, it turns out, because his employer wanted it that way. Vital details about how best to translate the

algorithm, whose mathematical notations run on for about 20 printed pages, into digital computer code were withheld to give Bell Labs a head start at developing commercial products. Following the breakup of American Telephone & Telegraph Co. in January, 1984, Bell Labs was no longer prevented from exploiting its research for profit. While the underlying concept could not be patented or copyrighted because it is pure knowledge, any computer programs that AT&T developed to implement the procedure can be protected.

Now, AT&T may soon be selling the first product based on Karmarkar's work—to the U.S. Air Force. It includes a multiprocessor computer from Alliant Computer Systems Corp. and a software version of Karmarkar's algorithm that has been optimized for high-speed parallel processing. The system would be installed at St. Louis' Scott Air Force Base, headquarters of the Military Airlift Command (MAC). Neither party will comment on the deal's cost or where the negotiations stand, but the Air Force's interest is easy to fathom.

**JUGGLING ACT.** On a typical day thousands of planes ferry cargo and passengers among air fields scattered around the world. To keep those jets flying, MAC



KARMARKAR: SKEPTICS ATTACKED HIS PRECOCIOUS FEAT

linear programming (LP) has evolved, and most scientists thought that was as far as they could go. Sure enough, when other researchers independently tried to test Karmarkar's process, their results were disappointing. At scientific conferences skeptics attacked the algorithm's validity as well as Karmarkar's veracity.

But this story may end with a different

- ▶ Obrigado pela atenção!
- ▶ Dúvidas?